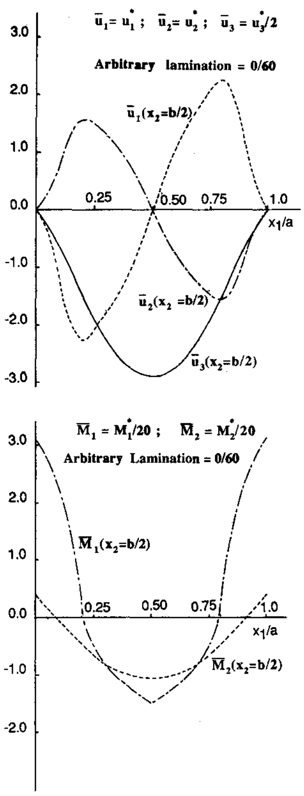

a) b)

Fig. 3 Variation of the normalized displacements and moments of a square moderately thick ($a/h = 10$) arbitrarily laminated (0 deg/60 deg) plate along the centerline, $x_2 = b/2$: a) displacements; b) moments.

expected, assume their maximum values at the center and edge of the plate, respectively, where in-plane displacements u_1^* and u_2^* vanish. The in-plane displacements u_1^* and u_2^* attain their maximum values near $x_1/a = 0.2$ and 0.8, where M_1^* vanishes. Moment M_2^* attains its maximum magnitude at the center of the plate.

Summary and Conclusions

A novel generalization of the almost two centuries-old Navier's approach is extended to obtain a boundary-continuous-displacement type analytical or strong (differential) form of solution to the hitherto unsolved problem of bending of moderately thick rectangular arbitrarily laminated plates with the rigidly clamped boundary conditions, prescribed at all four edges. The assumed solution functions are in the form of double Fourier sine series, which satisfy the rigidly clamped boundary conditions a priori in a manner similar to Navier's method. The convergence characteristics of the response quantities of interest demonstrate the computational efficiency of the approach. Most interestingly, the bending-stretching type coupling effect has been found to compensate, to a certain extent, the effect of transverse shear deformation. The numerical results presented herein are expected to contribute to achievement of optimal design through composite tailoring.

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Buckling Testing of Composite Columns

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Introduction

THE objective of this Note is to report global (Euler) buckling testing of large composite columns.[1,2] Mass production of composite structural members (e.g., by pultrusion) makes composite materials cost competitive with conventional ones. In the pultrusion process, fibers impregnated with a polymer resin are pulled through a heated die that provides the shape of the cross section to the final product. Pultrusion is a continuous process for manufacturing prismatic sections of virtually any shape,[3] but mainly open or closed thin-walled cross sections. For long composite columns, overall (Euler) buckling is more likely to occur before any other instability failure. For short columns, local buckling occurs first, leading either to large deflections and finally overall buckling, or to material degradation due to large deflections (crippling). The local buckling critical load, determined using a plate analysis,[4] is used in this investigation to limit the long-column region. Because of the large elongation to failure allowed by both the fibers (e.g., 4.8%) and the resin (e.g., 4%), the composite material remains linearly elastic for large deflections and strains, unlike conventional materials that yield (steel) or crack (concrete) for moderate strains. Therefore, buckling is the governing failure for this type of cross section, and the critical buckling load is directly related to the load carrying capacity of the member.

The classical Euler theory[5] used for the buckling of slender columns of isotropic materials reduces the instability problem to a matter of geometry. A similar analysis for pultruded composite columns concludes with the determination of the effective bending stiffness of the cross section, which accounts for the varying material properties. The classical analysis follows Tsai[6] and Vinson[7] but must be complemented by the appropriate plane stress assumption through the width of the beam.[8,9] The material properties of a pultruded column can be accurately predicted from the description of the cross section used in the manufacturing process.[9] Euler's theory assumes an initially straight column with no eccentricity or imperfections, such as initial crookedness. Agreement between the critical load obtained in laboratory experiments and the critical load determined by Euler's analysis is a somewhat fortuitous occurrence and is expected only in the case of perfect columns.

Southwell[10,11] accounted for this inconsistency by using a data reduction technique on the hyperbolic experimental data. In Southwell's method, the critical load is determined by using

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the asymptote of the experimental measurements. The method is attractive because it does not require one to reach the critical load, it is nondestructive, and it properly accounts for imperfections in the column or the testing fixture. Southwell's method was extended to account for extreme eccentricities[12] and transverse lateral loads.[13] The method is ideal for composite materials that remain linear for large values of strains. Since significant imperfections exist in pultruded composite columns because of the nature of the manufacturing process, experimental results can be expected to behave in a manner similar to Southwell's tests. The bending stiffness used to predict the Euler load can be predicted by the analysis presented here and verified by bending tests.[14,15] However, the loading conditions of a bending test subjects the material to a state of stress and deformation that is entirely different from the column loading. Furthermore, buckling tests may reveal the occurrence of local buckling, even for long columns.[16] Therefore, there is a need to evaluate experimentally the column buckling load of composite structural shapes.

Theoretical Analysis

For thin-walled columns with some laminas parallel and some perpendicular to the plane of bending, the bending stiffness of the column can be computed as

$$D = \left[D_{11} + \frac{2D_{16}D_{26}D_{12} - D_{66}D_{12}^2 - D_{22}D_{16}^2}{D_{22}D_{66} - D_{26}^2} \right] w \tag{1}$$

where the width of the flange w is introduced to obtain a beam bending stiffness with the same units of EI. The material properties (E_1, E_2, G_{12}, D_{12}) for each layer can be calculated from the description of the cross section used for manufacturing.[3,9] Hence, the buckling load for a pinned-pinned composite column is

$$N_{xcr} = \frac{\pi^2 D}{L^2 b} \quad \text{or} \quad P_{cr} = \frac{\pi^2 D}{L^2} \tag{2}$$

Computation of the bending coefficients in Eq. (1) is based on the micromechanical data[3] for each wide flange I-beam section being tested. A complete description of the stacking sequence and micromechanical computations for all layers in all sections being tested are presented by Tomblin.[17] Using Eq. (2), the Euler buckling load is computed for each I-beam section and reported in Table 1.

Thin-walled members may twist as they buckle due to an axially compressive load.[4] An approximate analysis was performed to verify that torsional buckling will not occur while testing for long-column buckling. The most extreme case is when $L \to 0$ and simple (torsional) supports are used on a strong axis test. The computed ratio of the critical loads is found to be larger than 2.0 for all sections tested, which indicates that the torsional buckling load is at least twice the Euler buckling load about the strong axis. In the testing frame used in this investigation for the long-column tests, the torsional supports are fixed because the specimens are placed 102 mm (4 in.) inside a steel shoe in which no twisting can occur.

Table 1 Experimental Euler buckling loads

Section, mm	Length, m	$P_{cr\ theory}$, kN	$P_{cr\ exp}$, kN	Difference, %
102 × 102 × 6.4[a]	4.48	12.46	12.08	3.12
102 × 102 × 6.4[a]	2.98	28.11	27.21	3.22
152 × 152 × 6.4[a]	6.03	23.66	23.10	2.30
152 × 152 × 6.4[a]	3.58	67.11	64.15	4.40
152 × 152 × 9.5[a]	6.03	34.11	33.38	2.14
152 × 152 × 9.5[a]	3.89	82.22	78.80	5.88
102 × 102 × 6.4[b]	4.48	42.28	40.07	5.22
152 × 152 × 9.5[c]	6.03	11.56	11.12	3.86

[a]Weak axis, no eccentricity. [b]Strong axis, no eccentricity.
[c]Strong axis with eccentricity.

Also, the column lengths that were tested had a length greater than 1.22 m (4 ft). As shown by Tomblin,[17] this will increase the ratio of the critical loads and decrease the possibility of twisting. Therefore, when testing the weak axis of the specimens, a factor of approximately 9.0 existed between the torsional critical load and the bending critical load. Hence, the possibility of any twisting occurring in the columns during testing is minimal.

Experimental Setup and Testing Procedure

Southwell's method[10,11] was used in this investigation to determine the critical load from the experimental data. The method works well when there is no mode interaction and the imperfections have a strong component of the form of the buckling mode. Load vs lateral deflection is recorded, and, if the buckling mode is isolated, the data show a hyperbolic shape. The raw data are transformed to obtain the asymptote of the hyperbola from a linear regression of the transformed data. All measurements were taken from a central point with respect to the column length. However, as shown by Tomblin,[17] any point along the length can be used to measure the deflections. Enough data points must be collected in the linear range of the material to obtain a good regression. This limits the applicability of the method for the case of almost perfect metal columns[12,13] but not for composite columns because of the large elongation to failure of the material.

The Euler or long-column tests were performed using a testing machine specifically developed for this investigation (Fig. 1). The testing frame can accommodate up to 30.5 × 30.5 cm (12 × 12 in.) cross sections, with column lengths ranging from 1.22 to 6.10 m (4 to 20 ft). A Materials Testing System (MTS) hydraulic actuator was used to apply a compressive load onto the composite specimens. All specimens were manufactured with pultruded columns supplied by Creative Pultrusions, Inc. Long-column tests were performed on the following I-beams: 102 × 102 × 6.4 mm, 152 × 152 × 6.4 mm, and 152 × 152 × 9.5 mm (4 × 4 × 1/4 in., 6 × 6 × 1/4 in., and 6 × 6 × 3/8 in.: depth, width, and thickness).

The testing frame is fitted with three steel crossheads (one fixed, one movable, and one adjustable) mounted to two 30-ft-long channels (Fig. 1). A shoe (or grip) is mounted to each of the two interior crossheads by a pin and needle bearing that supplies a pinned-pinned end condition to the column specimen. Buckling is restricted to the plane of bending defined by the pinned shoes. A 50.8-cm (20 in.) linear variable differential transducer (LVDT) was mounted to the center of the specimen to obtain a center deflection of the column. A load cell was used between the actuator and the movable crosshead to record the applied compressive load. A second LVDT was mounted to the actuator to measure and control the axial displacement. All tests are performed under "displacement control," with the axial displacement being controlled by the closed-loop servo-control system. Various limits

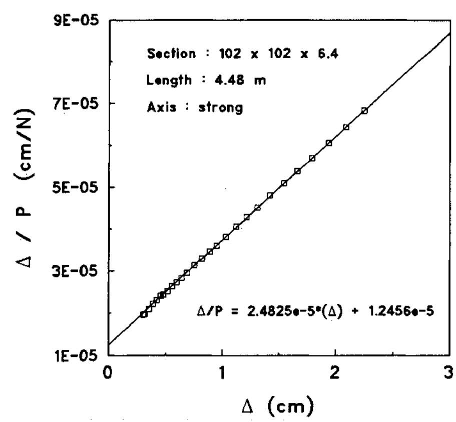

Fig. 1 Buckling testing frame.

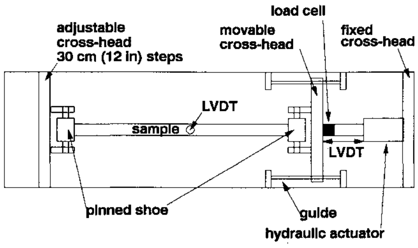

Fig. 2 Linearized Southwell plot for the 102 × 102 × 6.4 mm (4 × 4 × 1/4 in.) I-beam (length = 4.48 m) buckling about the strong axis, without load eccentricity.

for loads and displacements were programmed into the control system to provide a safe operation.

The testing procedure consisted of loading the column using displacement control to regulate the MTS actuator. After some initial testing, it was decided that, to obtain reasonably good results, a center deflection of $L/100$ (where L is the column length) should be approached. This is necessary to have a sufficiently large number of points in the Southwell plot while keeping the material in the linear range. As the test was performed, LVDT and load cell reading data were recorded using a METRABYTE Data Acquisition board in an IBM compatible PC running LABTECH Notebook data acquisition software. At least two tests were done on each column length selected. The beam was flipped over (180 deg) to obtain two independent data sets about the same bending axis in an attempt to cancel out any fixture misalignment. No significant differences were apparent. Teflon-coated lateral supports were also required to restrain the column from buckling in the fixed-fixed mode (about the weak axis) when testing the strong axis of the specimen. All specimens were loaded and measurements taken until the factor $L/100$ was approached. At this point, the test was stopped and the column was returned to the initial position.

Experimental Results

The lengths selected for testing are within the long-column range, with no possibility of branching into or interacting with the short-column modes.[5,9] During the test, a data acquisition system collected load P and central deflection Δ data. The hyperbolic Δ-P results were then linearized on a Δ-γ plot, where $\gamma = \Delta/P$. A linear regression was done, and the resulting slope was used to obtain the buckling load (Fig. 2). Southwell tests on the weak and strong axis were performed on the pultruded I-beam sections listed in the preceding section. At least two lengths in the long-column range were tested for each section. After the first test, the beam was rotated 180 deg and a second test was performed. On several occasions, more than two tests were done on each length, taking advantage of the nondestructive nature of the test.

Figure 2 shows the linear regression of the data acquired during the strong axis testing of a 102 × 102 × 6.4 mm (4 × 4 × 1/4 in.) section loaded without eccentricity. Table 1 shows the length, theoretical buckling load, experimental buckling load (inverse of slope), and the percentage difference between the theoretical and experimental loads for each section tested. The percentage error is within 6% for all sections tested, and all loads were below the theoretical prediction. From the results, one can conclude that the theoretical load can be viewed as the upper bound for the actual buckling load. Since the theoretical prediction assumes perfect manufacturing and layup conditions (no voids, imperfections, etc.), this is a reasonable result because of unavoidable imperfections in the manufacturing process.

Because of the limitation of the MTS actuator to 55,000 lb, the strong axis buckling load of several sections could not be approached. In this case, the load was applied with an eccentricity at the column ends. This eccentricity produced a bending moment in addition to the axial load in order to lower the required load needed to produce a center deflection of $L/100$. In Southwell's method, real imperfections and load eccentricity are both taken into account as an equivalent imperfection. The intercept of the regression line with the Δ axis is, therefore, large because of the 1 in. eccentricity of loading. In this case, a number of data points are neglected for the regression analysis.[17]

Conclusions

The experimental procedure produces highly reproducible and accurate results. All percentage differences between the theoretical and experimental are below 6.2%. The theoretically predicted (Euler) long-column buckling load is accurate even for complex materials considered in this investigation.

Acknowledgments

The financial support of the Transportation Systems Rehabilitation and Maintenance Institute at West Virginia University is appreciated. Special recognition is given to Creative Pultrusions, Inc., which provided financial support and the samples for the experimental program.

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